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$$\text{and } \alpha+\beta=\frac{2v_1w_1-2v_2w_2}{(v_1+w_1)-(v_2+w_2)}. \tag{10}$$

From these equations it is easily seen that

$$2\alpha\beta-(\alpha+\beta)(v_1+w_1)=-2v_1w_1,$$
$$\text{and } 2\alpha\beta-(\alpha+\beta)(v_2+w_2)=-2v_2w_2.$$

But these are the characteristic equations for harmonic pairs. Therefore, the pair α, β is harmonic with respect to the pairs v_1, w_1 and v_2, w_2.

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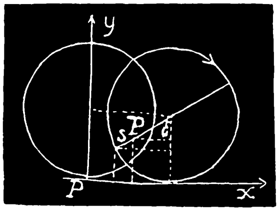
ANOTHER WAY TO GENERATE THE CYCLOID.

By **H. SCHAPPER**, Fayetteville, Ark.

Consider a point describing a simple harmonic motion along the diameter of a uniformly rolling circle, and let both motions begin simultaneously. If r be the radius of the circle, and ϵ the angle of rolling, and if a complete unrolling of the circle correspond to a full period of the simple harmonic motion, then we get

$$s=r(1-\cos\epsilon)$$



as the equation for the simple harmonic motion.

From the figure we see that

$$x=r(\epsilon-\sin\epsilon)+r(1-\cos\epsilon)\sin\epsilon,$$
$$y=r(1-\cos\epsilon)+r(1-\cos\epsilon)\cos\epsilon.$$

After an easy reduction we get

$$x=r(\epsilon-\tfrac{1}{2}\sin 2\epsilon),\quad y=r\sin^2\epsilon.$$

These equations may be written in the following way, or

$$x=\frac{r}{2}(2\epsilon-\sin 2\epsilon),\quad y=\frac{r}{2}(1-\cos 2\epsilon),$$

or

$$x=R(\phi-\sin\phi),\quad y=R(1-\cos\phi),$$

which is the parametric form of the equations of the cycloid. The cycloid thus generated is the same as if described by a simple rolling of a circle of radius $\frac{1}{2}r$. From

$$\frac{dy}{dx}=\frac{\sin 2\epsilon}{(1-\cos 2\epsilon)}=\cot\epsilon=\tan(\tfrac{1}{2}\pi-\epsilon),$$

it follows that the diameter $2r$ of the circle is always tangent to the curve, so that we get the points of the curve as well as their corresponding tangents at the same time, which is rather an advantage. The construction of the cycloid is thus simplified. Besides, it is of interest that the definition of the cycloid is thus given in a different way from the one commonly employed.

DEPARTMENTS.

PROBLEMS FOR SOLUTION.

ALGEBRA.

305. Proposed by S. A. COREY, Hiteman, Iowa.

Prove or disprove, that $\sum\limits_{n=1}^{n=\infty}\frac{1}{(2n-1)^2+4}=\frac{\pi}{4}-\frac{\pi}{8}\left(\frac{\cosh\pi}{\sinh\pi}\right)$.

Solution by J. SCHEFFER, A. M., Kee Mar College, Hagerstown, Md.

Taking the logarithm of the well known formula

$$\cos\tfrac{1}{2}\pi x=\left(1-\frac{x^2}{1}\right)\left(1-\frac{x^2}{9}\right)\left(1-\frac{x^2}{25}\right)\dots$$

and differentiating, we have

$$\tfrac{1}{2}\pi\tan\tfrac{1}{2}\pi x=\frac{2x}{1-x^2}+\frac{2x}{9-x^2}+\frac{2x}{25-x^2}+\dots$$

Putting $x\sqrt{-1}$ in place of x, we have

$$\tfrac{1}{2}\pi\tan(\tfrac{1}{2}\pi x\sqrt{-1})=2x\sqrt{-1}\left[\frac{1}{1+x^2}+\frac{1}{9+x^2}+\frac{1}{25+x^2}+\dots\right]$$

Putting $x=2$, we have